WJEC (Eduqas) Physics A-level

Topic 1.5: Solids Under Stress

Notes

Hooke's Law

Hooke's law states that for an elastic spring, the extension is directly proportional to the force applied to its end.

$$F = kx$$

- F is the **force** applied
- k is the **spring constant** or force per unit length
- x is the **extension**

Having a larger spring constant means a larger force needs to be applied to get the same extension and therefore the spring is **stiffer**. A lower spring constant means a less stiff spring.

Stress and Strain

The measurements of force and extension are useful for comparisons for samples of only the **same material** because the spring constant will vary depending on original length and cross-sectional area. If you want to compare different materials, you need to consider this variation.

We will introduce two new quantities called stress and strain.

$$stress = \frac{force}{cross\text{-}sectional\ area}$$

$$\sigma = \frac{F}{A}$$

$$strain = \frac{extension}{original\ length}$$

$$\varepsilon = \frac{\Delta l}{l}$$

These are **both tensile properties** meaning they are used when a material is in tension.

From these we can derive a new **property of a material called the Young modulus** (*E)* which is like the spring constant except the Young modulus is constant for a material. Two samples (of the same material may have different spring constants but will have the same Young modulus).

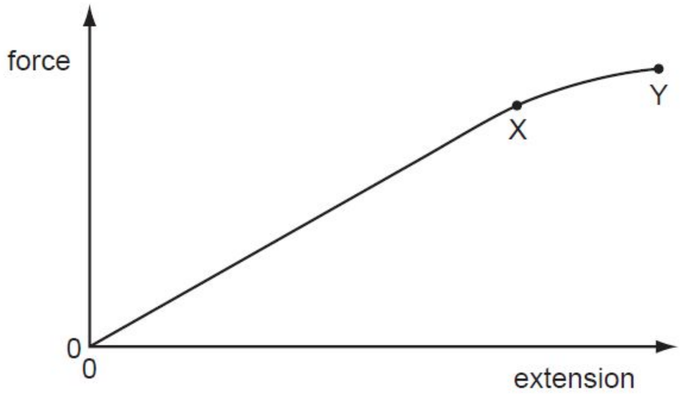
The Young modulus is found by taking the ratio of stress to strain **when the sample is obeying Hooke's law**:

$$E = \frac{\sigma}{\varepsilon}$$

Force-Extension Graphs

You can plot the force and extension on a graph to figure out some properties.

A typical force-extension graph will look like the following,



The **area under the graph is equal to the work done** in extending the material. Up until the point X, the material is extending elastically which means it obeys Hooke's law and if the force was removed it would return to its original shape and length.

From point X to point Y, the **material extends plastically** which means force is no longer proportional to extension – Hooke's law does not apply – and the material **will not return to its original shape or length** when released.

For the straight portion of the graph (obeying Hooke's law) the work done in extending the material is equal to the average force multiplied by the total extension i.e.

$$W = \frac{1}{2}Fx$$

As Hooke's law is being applied we can say that $F = kx$ giving us,

$$W = \frac{1}{2}(kx)x$$

$$W = \frac{1}{2}kx^2$$

This is the elastic potential energy equation (shown in topic 1.4) and is seen here because as work is done extending the material elastically, the energy is transferred into elastic potential energy and is stored.

When the material extends beyond the Hooke's law region (called the elastic region) the additional work done is used to break bonds in the material to permanently deform it and therefore we cannot say that the work done is to increase elastic potential energy.

In fact, none of the above equations are now valid. The average force is not now half the maximum force and therefore we cannot use the equations we derived using this. However, the work done is still equal to the area under the graph.

Classification of Solids

A solid may be classified into three categories:

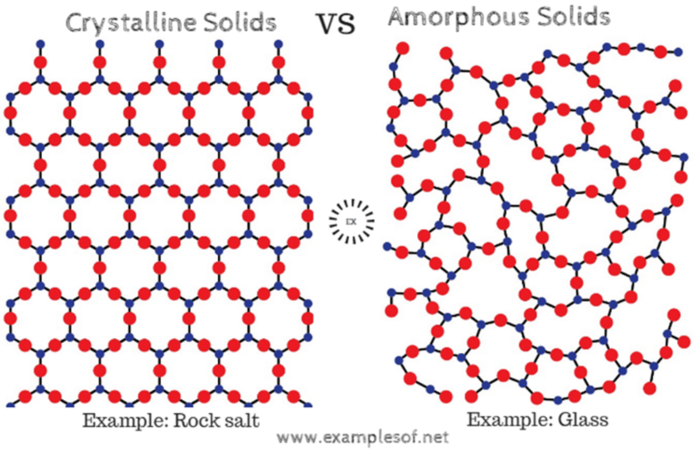
- Crystalline
- Amorphous
- Polymeric

Crystalline

These are materials which have atoms arranged in an **ordered structure making a crystal lattice**. Most metals are crystalline and extend elastically for small strains. A diagram showing the regular structure is shown below. The atoms in metals are ionised and the positive ions form a regular lattice and can slip meaning metals are **ductile** (can be drawn out into a wire) and **tough** (absorb a lot of energy before breaking).

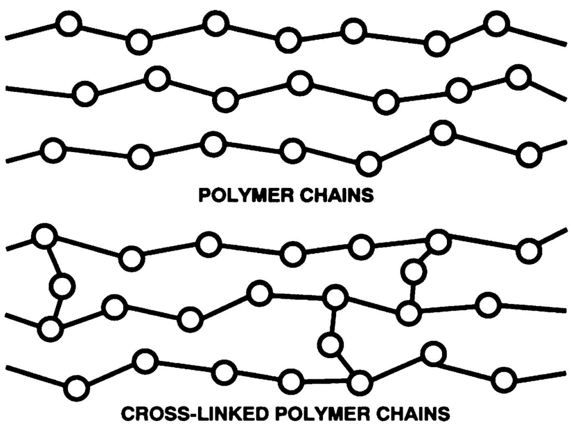
Amorphous

These are materials where the atoms or molecules are arranged in a non-ordered structure. Examples of amorphous materials are glasses and ceramics. Ceramics have **rigid structures** as they are formed by directional covalent bonds locking atoms into place.



Polymeric

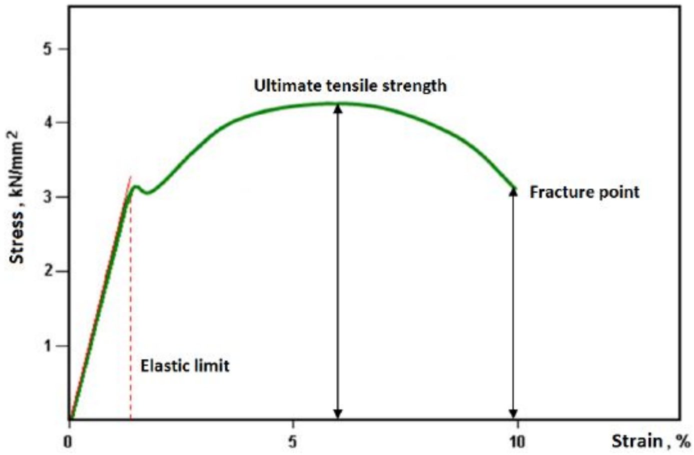
These are materials that contain long chain polymer molecules. The bonds in most polymers are strong and can rotate. Therefore, most polymers are **strong and flexible**. Cross-links (where chains are tied together) can be added to polymer chains to make them stiffer.



Stress-Strain Graphs

The Young modulus is given by the **gradient of the straight-line** section of the stress-strain graph.

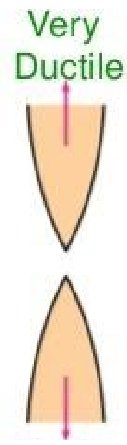
Metals



This graph shows that metals can **plastically deform for a long time** before they break (or fracture). They absorb a lot of energy before this occurs. The Young modulus is high as metals are very strong.

Dislocations are small gaps in the ordered structure of crystalline materials and they allow planes of atoms to slip more easily as one row of atoms can slip at a time as the dislocation is passed through the metal. This makes metals weaker.

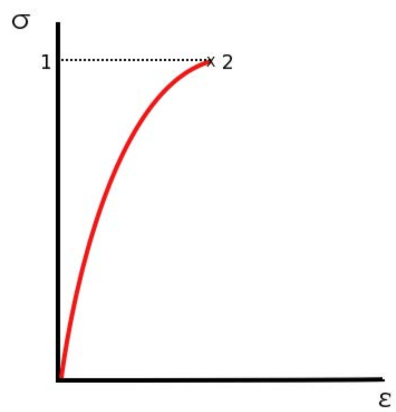
To reduce the effect of these dislocations you can introduce **'foreign atoms'**. These are atoms which fill the gaps to prevent them from moving. Another method is to create **more grain boundaries**. This means there are fewer dislocations per grain and so it becomes harder for dislocations to move across boundaries.



Necking is where the cross-sectional area of the metal reduces as it deforms plastically. To keep extending the metal and lower stress is needed shown by the dip in the curve on the stress-strain graph.

Ductile fracture occurs when necking continues until the material separates at a point (shown in the diagram below).

Brittle Materials



Brittle materials are often very strong and so have a high Young modulus. However, they are not tough and so they do not absorb much energy at all before fracturing.

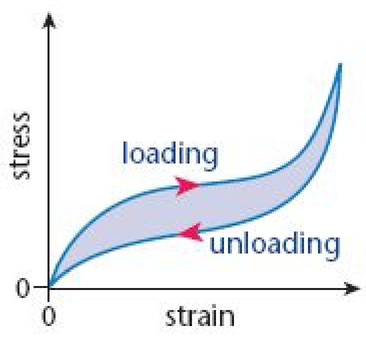
We can see that the brittle material obeys Hooke's law almost until fracture.

Crack propagation often causes brittle fracture. This is when there are small **imperfections** on the surface on a material which concentrate the stress on the surface and cause a crack to open and therefore causing the material to fracture. It will undergo brittle fracture - as it is a crack, the material will hardly extend before it breaks.

These surface imperfections will reduce the breaking stress of a material because it will allow for cracks to develop.

If you put the surface under compression, then a much larger stress is required to separate the two sides of the crack and to get the rest of the material into tension, so it can separate.

Rubber



Hooke's law is only approximately obeyed for rubber and it has a **low Young modulus**, meaning it is not very stiff.

When you stretch rubber, the long chain polymer molecules it contains begin all tangled up and gradually line up together until they're approximately straight. You can see that at first, it is more difficult to stretch the rubber, then it becomes easier in the middle of the graph and then it becomes harder again.

Hysteresis is when the stress-strain graph is different depending on whether you are loading or unloading the material. The area between the two curves is equal to the energy per unit volume stored as thermal energy in the rubber after it has been loaded and then unloaded.